

> restart;

> # Find Adams-Bashforth method of order s

> s:=4;

$$s := 4$$

> t:=n->t\_0+n\*h;

$$t := n \rightarrow t_0 + n h$$

> L:=k->product((tau-t(n-j))/(t(n-k)-t(n-j)),j=0..k-1)  
\*product((tau-t(n-j))/(t(n-k)-t(n-j)),j=k+1..s-1);

$$L := k \rightarrow \left( \prod_{j=0}^{k-1} \frac{\tau - t(n-j)}{t(n-k) - t(n-j)} \right) \left( \prod_{j=k+1}^{s-1} \frac{\tau - t(n-j)}{t(n-k) - t(n-j)} \right)$$

> p:=0:

for i from 0 to s-1

do

p:=p+simplify(L(i))\*f[n-i];

od:

p:=collect(p,tau);

$$p := \left( \frac{f_n}{6 h^3} - \frac{f_{n-1}}{2 h^3} + \frac{f_{n-2}}{2 h^3} - \frac{f_{n-3}}{6 h^3} \right) \tau^3 + \left( - \frac{(3 t_0 + 3 n h - 6 h) f_n}{6 h^3} \right. \\ + \frac{(3 t_0 + 3 n h - 5 h) f_{n-1}}{2 h^3} - \frac{(3 t_0 + 3 n h - 4 h) f_{n-2}}{2 h^3} \\ + \left. \frac{(3 t_0 + 3 n h - 3 h) f_{n-3}}{6 h^3} \right) \tau^2 + \left( \frac{-(t_0 + n h - h)(t_0 + n h - 2 h) + (-2 t_0 - 2 n h + 3 h)(t_0 + n h - 3 h) f_n}{6 h^3} \right. \\ + \frac{-(t_0 + n h)(t_0 + n h - 2 h) + (-2 t_0 - 2 n h + 2 h)(t_0 + n h - 3 h) f_{n-1}}{2 h^3} \\ - \frac{-(t_0 + n h)(t_0 + n h - h) + (-2 t_0 - 2 n h + h)(t_0 + n h - 3 h) f_{n-2}}{2 h^3} \\ + \left. \frac{-(t_0 + n h)(t_0 + n h - h) + (-2 t_0 - 2 n h + h)(t_0 + n h - 2 h) f_{n-3}}{6 h^3} \right) \tau \\ - \frac{(t_0 + n h - h)(t_0 + n h - 2 h)(t_0 + n h - 3 h) f_n}{6 h^3}$$

$$\begin{aligned}
& + \frac{(t_0 + n h) (t_0 + n h - 2 h) (t_0 + n h - 3 h) f_{n-1}}{2 h^3} \\
& - \frac{(t_0 + n h) (t_0 + n h - h) (t_0 + n h - 3 h) f_{n-2}}{2 h^3} \\
& + \frac{(t_0 + n h) (t_0 + n h - h) (t_0 + n h - 2 h) f_{n-3}}{6 h^3}
\end{aligned}$$

> **AB:=simplify(int(p,tau=t(n)..t(n+1)));**

$$AB := \frac{1}{24} h (55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3})$$

> **y[n+1]=y[n]+AB;**

$$y_{n+1} = y_n + \frac{1}{24} h (55 f_n - 59 f_{n-1} + 37 f_{n-2} - 9 f_{n-3})$$

>