```
> restart;
> # Find Gaussian Quadratue on [a,b] with weight function w
  #
  # Note: If the weight function is complicated, then the integrals
  # in the Gram-Schmidt algorithm may not have closed form solutions.
  # In this case insert evalf( ... ) in suitable places to compute
  # the coeficients numerically.
> with(LinearAlgebra):
> n:=3;
```

$$n := 3$$

```
> a:=0;
  b:=1;
  w:=x->1;
```

$$a := 0$$

$$b := 1$$

$$w := x \rightarrow 1$$

```
> for k from 0 to n
  do
    q[k]:=x^k;
    for j from 0 to k-1
    do
      q[k]:=q[k]-q[j]*int(q[k]*q[j]*w(x),x=a..b);
    end:
    q[k]:=expand(q[k]/sqrt(int(q[k]^2*w(x),x=a..b)));
  end:
> p:=q[n];
```

$$p := 20\sqrt{7}\,x^3 - \sqrt{7} + 12\sqrt{7}\,x - 30\sqrt{7}\,x^2$$

```
> c:=Vector([solve(p=0)]);
```

$$c := \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} + \frac{1}{10}\sqrt{15} \\ \frac{1}{2} - \frac{1}{10}\sqrt{15} \end{bmatrix}$$

```
> V:=VandermondeMatrix(c);
```

$$V := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{4} \\ 1 & \frac{1}{2} + \frac{1}{10}\sqrt{15} & \left(\frac{1}{2} + \frac{1}{10}\sqrt{15}\right)^2 \\ 1 & \frac{1}{2} - \frac{1}{10}\sqrt{15} & \left(\frac{1}{2} - \frac{1}{10}\sqrt{15}\right)^2 \end{bmatrix}$$

```
> Y:=Vector([seq(int(x^k*w(x),x=a..b),k=0..n-1)]);
```

$$Y := \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}$$

```
> b:=LinearSolve(Transpose(V),Y);
```

$$b := \begin{bmatrix} \frac{4}{9} \\ \frac{5}{18} \\ \frac{5}{18} \end{bmatrix}$$

```
> F:=unapply(DotProduct(b,Map(f,Vector(c))),f);
```

$$F := f \rightarrow \frac{4}{9} f\left(\frac{1}{2}\right) + \frac{5}{18} f\left(\frac{1}{2} + \frac{1}{10}\sqrt{15}\right) + \frac{5}{18} f\left(\frac{1}{2} - \frac{1}{10}\sqrt{15}\right)$$

```
> F(x->1);
  simplify(F(x->x));
  simplify(F(x->x^2));
  simplify(F(x->x^3));
  simplify(F(x->x^4));
  simplify(F(x->x^5));
  simplify(F(x->x^6));
```

$$1$$

$$\frac{1}{2}$$

$$\frac{1}{3}$$

$$\frac{1}{4}$$

$$\frac{1}{5}$$

$$\frac{1}{6}$$

$$\frac{57}{400}$$