

```
> restart;
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```
> with(LinearAlgebra):
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```
> m:=4;
```

$m := 4$

```
> A:=Matrix([[0,0],[1/3,1/3]]);
```

$$A := \begin{bmatrix} 0 & 0 \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

```
> b:=Vector([1/4,3/4]);
```

$$b := \begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$$

```
> c:=Vector([0,2/3]);
```

$$c := \begin{bmatrix} 0 \\ \frac{2}{3} \end{bmatrix}$$

```
> n:=Dimension(b);
```

$n := 2$

```
> onesvector:=Vector(n,1);
```

```
fxi:=Vector([seq(f(t+c[i]*h,xi[i]),i=1..n)]);
```

```
xirhs:=y(t)*onesvector+h*Multiply(A,fxi);
```

$$\text{onesvector} := \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$fxi := \begin{bmatrix} f(t, \xi_1) \\ f\left(t + \frac{2}{3}h, \xi_2\right) \end{bmatrix}$$

$$xirhs := \begin{bmatrix} y(t) \\ y(t) + h \left(\frac{1}{3} f(t, \xi_1) + \frac{1}{3} f\left(t + \frac{2}{3}h, \xi_2\right) \right) \end{bmatrix}$$

```
> T:=y(t+h)-y(t)-h*Multiply(Transpose(b),fxi);
```

```

for j from 1 to m-1
do
  T:=subs(xi=xirhs,T);
od:

```

$$T := y(t+h) - y(t) - h \left(\frac{1}{4} f(t, \xi_1) + \frac{3}{4} f\left(t + \frac{2}{3}h, \xi_2\right) \right)$$

```

> S:=series(T,h,m);

```

$$\begin{aligned}
S := & (-f(t, y(t)) + D(y)(t) h + \left(-\frac{1}{2} D_1(f(t, y(t))) - \frac{1}{2} D_2(f(t, y(t))) f(t, y(t)) + \frac{1}{2} D^{(2)}(y)(t) \right) h^2 + \\
& - \frac{1}{6} D_2(f(t, y(t)))^2 f(t, y(t)) - \frac{1}{6} f(t, y(t))^2 D_{2,2}(f(t, y(t))) - \frac{1}{6} D_2(f(t, y(t))) D_1(f(t, y(t))) \\
& - \frac{1}{6} D_{1,1}(f(t, y(t))) - \frac{1}{3} D_{1,2}(f(t, y(t))) f(t, y(t)) + \frac{1}{6} D^{(3)}(y)(t) \Big) h^3 + O(h^4)
\end{aligned}$$

```

> eq[1]:=D(t->y(t))(t)=f(t,y(t));

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$$eq_1 := D(y)(t) = f(t, y(t))$$

```

> for j from 1 to m-2
do

```

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  eq[j+1]:=simplify(subs(seq(eq[i],i=1..j),D(unapply(eq[j],t))(t)));

```

```

od;

```

$$eq_2 := D^{(2)}(y)(t) = D_1(f(t, y(t))) + D_2(f(t, y(t))) f(t, y(t))$$

$$\begin{aligned}
eq_3 := & D^{(3)}(y)(t) = D_{1,1}(f(t, y(t))) + 2 D_{1,2}(f(t, y(t))) f(t, y(t)) + f(t, y(t))^2 D_{2,2}(f(t, y(t))) \\
& + D_2(f(t, y(t))) D_1(f(t, y(t))) + D_2(f(t, y(t)))^2 f(t, y(t))
\end{aligned}$$

```

> simplify(subs(seq(eq[i],i=1..m-1),S));

```

$$O(h^4)$$