Course Summary for MATH/CS 467/667

Note: Topics identified with • will not be on the Math/CS 467 final exam; however, these topics may appear on the test for Math/CS 667 and as extra credit for Math/CS 467.

1. Euler's Method and Beyond
   i. Definition of the Lipschitz Condition.
   ii. Precise statement of Taylor's Theorem.
   iii. Euler's method and the Backwords Euler's Method.
   iv. How to sum a geometric series.
   v. Proof that Euler's method is convergent.
   vi. Definition of truncation error.
   vii. How to compute truncation error for the trapeziodal rule and the theta method.
2. Multistep Methods
   i. The general form of an s-step method.
   ii. The idea (in terms of interpolating polynomials) behind the Adams–Bashforth and Adams–Moulton multistep methods.
   iii. Definition of the polynomials $\rho(\omega)$ and $\sigma(\omega)$.
   iv. What it means for $\rho(\omega)$ to satisfy the root condition.
   v. Be able to check the root condition for simple polynomials.
   vi. Statement of the Dahlquist equivalence theorem.
   vii. How to find the order (as in Theorem 2.1) of a multistep method in terms of the polynomials ρ and σ.
   viii. The definition of a backward differentiation formula, i.e., that $\sigma(\omega) = \beta\omega^s$ for some $\beta \in \mathbf{R} \setminus \{0\}$.
   ix. Statement of the Dahlquist first barrier.
3. Gaussian Quadrature
   i. The way to derive Gaussian quadrature formulae.
   ii. Proof of Lemma 3.2.
   iii. Proof of Theorem 3.3(i) that the Gaussian quadrature formula is order 2ν.
4. Runge–Kutta Methods
   i. Translation of the RK tableaux in terms of algebraic equations that could be programmed into a computer.
   ii. How to tell (by looking at the matrix A) whether a given RK method is explicit.
   iii. The relationship between IRK methods and collocation as stated in Lemma 3.5.
   iv. •Theorem 3.7 and its Corollary.
5. Stiff ODEs
   i. What is the linear stability domain $\mathcal{D}$?
   ii. Be able to find $\mathcal{D}$ for Euler's method, the implicit Euler's method.
   iii. Definition of A-stability.
   iv. What is $r(z)$ for RK methods?
   v. Proof that no explicit RK method is A stable.
   vi. Definition that $r(z)$ is order p.

vii. Definition of a Padé approximant.
viii. Statement of The Wanner–Hairer–Nørsett theorem.
ix. •Statement of the Dahlquist second barrier.

6. Error Control
   i. Describe the Milne Device.
   ii. Find the error control given by the Milne device for specific multistep pairs of the same order.
   iii. How can two methods of different order be used for error control?
   iv. What is the idea behind Embedded RK methods?
7. Nonlinear Algebraic Systems
   i. Explain how to use functional iteration to advance an implicit integration scheme.
   ii. What is the convergence criterion that functional iteration converges?
   iii. Explain the disadvantages of the functional iteration for stiff ODEs?
   iv. What is Newton's method?
   v. State the convergence criterion for Newton's method.