

Final Exam for Math/CS 467/667 given Spring 2008

**INSTRUCTIONS: Undergraduate students complete 3 out of the following 4 problems. Graduate students complete all 4. Clearly indicate which problems you wish graded.**

1. Explain why Euler's explicit method

$$y_{n+1} = y_n + hf(y_n, t_n)$$

should not be used to solve the initial value problem  $y' = f(y, t)$ ,  $y(0) = y_0$ . Suggest an alternative method and state the reasons why that method is better.

2. Find the truncation error in the 3-stage RK scheme

$$\begin{aligned}k_1 &= hf(y_n, t_n) \\k_2 &= hf(y_n + 2k_1/3, t_n + 2h/3) \\k_3 &= hf(y_n + k_2/3, t_n + h/3) \\y_{n+1} &= y_n - \frac{1}{8}(k_1 - 3k_2 - 6k_3)\end{aligned}$$

for solving the initial value problem  $y' = f(y, t)$ ,  $y(0) = y_0$ . What is the order of this method? Is this an efficient method in terms of order and number of stages?

3. Matlab has a builtin fast Fourier transform function called `fft`.

- (i) Check how fast it is by computing the time  $T_N$  to perform a transform of length  $N = 2^n$  for values of  $n$  ranging from 8 to as large as your computer can handle. Make each timing three times and take the smallest one for  $T_N$ . Note that the first timing is usually the longest because of initialization code that has to be run each time a transform of a different length is used. Plot a graph of  $T_N$  versus  $N$ . Is  $T_N$  closer to  $N \log N$  or  $N^2$ ? Explain.
- (ii) Repeat the above where  $N = p_i$  and  $p_i$  is a sequence of prime numbers. Note that a prime numbers can be found in Maple using the `isprime` command. In this case is  $T_N$  closer to  $N \log N$  or  $N^2$ ? Explain.

4. Consider the conservation law

$$u_t + \partial_x f(u) = 0 \quad \text{where } x \in [0, 2\pi] \text{ and } t \geq 0$$

with initial condition  $u(x, 0) = 2 + 3 \sin x$  and periodic boundary conditions. Draw an accurate graph of  $u(x, 1)$  corresponding to  $f(u) = \cos u$ .