Programming Assignment 2

Your work should be presented in the form of a typed report using clear and properly punctuated English. Where appropriate include full program listings and output. If you choose to work in a group of two, please turn in independently prepared reports.

4a. The Lorenz system is a three dimensional ordinary differential equation of the form

$$\frac{dy}{dt} = f(y)$$

with a given initial condition $y(0) = a$ where $y(t)$ is a vector in $\mathbf{R}^3$ and

$$f(y) = \begin{bmatrix} -10y_1 + 10y_2 \\ 28y_1 - y_2 - y_1y_3 \\ y_1y_2 - (8/3)y_3 \end{bmatrix}.$$

Let Y^n be an approximation of $y(1)$ obtained using a step size of $h = 1/n$. Define the error

$$E_n = \|Y^n - y(1)\| = \left\{ \sum_{i=1}^{3} \left(Y_i^n - y_i(1)\right)^2 \right\}^{1/2}.$$

Show that if $E_n \le Kh^k$ then

$$\|Y^n - Y^{2n}\| \le K\left\{1 + \frac{1}{2^k}\right\}h^k.$$

4b. Write a program to approximate solutions of the Lorenz system using Euler's forward difference method and the initial condition

$$a = \begin{bmatrix} 2 \\ 3 \\ 15 \end{bmatrix}.$$

Compute Y^n for $n = 64, 128, 256, 512, \ldots, 65536$.

4c. Graph $\log\|Y^n - Y^{2n}\|$ versus $\log h$ to verify the order of convergence for Euler's method numerically.

4d. Compute Y^n using Runge–Kutta methods of orders 2 and 4 and verify the order of convergence by graphing $\log\|Y^n - Y^{2n}\|$ versus $\log h$.

4e. Approximate $y(10)$ to three decimal places. Is it possible to achieve this accuracy using Euler's method? Can you find $y(100)$?