Math/CS 467/667 Programming Project 1

Rössler Oscillator

1. The Rössler System is a three dimensional ordinary differential equation of the form $dy/dt = f(y)$ with a given initial condition $y(t_0) = y_0$ where $y(t) \in \mathbf{R}^3$ and

$$f(y) = \begin{bmatrix} -y_2 - y_3 \\ y_1 + ay_2 \\ b + y_3(y_1 - c) \end{bmatrix} \qquad \text{where} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

with $a = b = 0.2$ and $c = 5.7$. Let Y^h be an approximation of $y(T)$ obtained using a step size of $h = (T - t_0)/n$. Define the error

$$E_h = \|Y^h - y(T)\| = \left\{ \sum_{i=1}^{3} \big(Y_i^h - y_i(T)\big)^2 \right\}^{1/2}.$$

Show that if $E_h \le Kh^k$ then

$$\|Y^h - Y^{h/2}\| \le K\Big\{1 + \frac{1}{2^k}\Big\}h^k.$$

2. Write a program to approximate $y(T)$ using Euler's forward difference method and the initial condition

$$y_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \text{when} \qquad t_0 = 0 \quad \text{and} \quad T = 1.$$

Compute Y^h for $h = T/n$ with $n = 64, 128, 256, 512, \ldots, 65536$.

3. Graph $\log \|Y^h - Y^{h/2}\|$ versus $\log h$ to verify the order of convergence for Euler's method numerically.

4. Computer Y^h using the Taylor methods of order 2 and 3 and verify the order of convergence by graphing $\log \|Y^h - Y^{h/2}\|$ versus $\log h$.

5. Approximate $y(10)$ to four decimal places. Indicate what method you used and how many steps were needed. Is it possible to achieve this accuracy using Euler's method? Can you find $y(100)$?