Math/CS 467/667 Sample Quiz 2 Version A

1. Answer one of the following two questions:

(i) Consider the trapezoid formula

$$T(\alpha, \beta, f) = \frac{f(\alpha) + f(\beta)}{2}(\beta - \alpha).$$

and the resulting quadrature method given by

$$\int_a^b f(x)dx \approx \sum_{j=0}^{N-1} T(x_j, x_{j+1}, f)$$

where $N \in \mathbf{N}$ and $x_j = a + hj$ with $h = (b - a)/N$. Write a program to approximate the integral

$$\int_0^{10} \frac{1}{2 + \sin x} dx$$

for $N = 10$ that prints the resulting approximation with 15 digits precision. Include the program and the output of the program in your submission.

(ii) Consider the differential equation given by

$$y' = \frac{y}{t} - \left(\frac{y}{t}\right)^2, \qquad y(1) = 1$$

Approximate the solution to this differential equation on the interval $[1, 2]$ using the RK4 method with $h = 1/20$. Print the approximation of $y(2)$ and the error $\big|y(2) - 2/(1 + \log 2)\big|$ to 15 digits precision. Include the program and the output of the program in your submission.

2. Answer one of the following two questions:

(i) Consider the 3-point Gaussian formula given by

$$G_3(\alpha,\beta,f)=\frac{\beta-\alpha}{2}\sum_{k=0}^{2} w_k f\Big(\alpha+\frac{\beta-\alpha}{2}(x_k+1)\Big)$$

where

$$\begin{array}{lll} x_0=-\sqrt{3/5}, & x_1=0, & x_2=\sqrt{3/5}, \\ w_0=5/9, & w_1=8/9, & w_2=5/9, \end{array}$$

and the resulting quadrature method given by

$$\int_a^b f(x)dx \approx \sum_{j=0}^{N-1} G_3(x_j,x_{j+1},f)$$

where $N\in\mathbf{N}$ and $x_j=a+hj$ with $h=(b-a)/N$. Write a program to approximate the integral

$$\int_0^5 \exp(\sin x)\,dx$$

for $N=10$ that prints the resulting approximation with 15 digits precision. Include the program and the output of the program in your submission.

(ii) Consider the differential equation given by

$$y'=\frac{1}{t^2}-\frac{y}{t}-y^2, \qquad y(1)=-1.$$

Approximate the solution to this differential equation on the interval $[1,2]$ using Taylor's 3rd order method with $h=1/30$. Print the approximation of $y(2)$ and the error $\big|y(2)+1/2\big|$ to 15 digits precision. Include the program and the output of the program in your submission.