Math/CS 467/667 Quiz 2 Version B

1. Answer one of the following two questions according to the following rule: If you plan to answer question i on page 2 then answer question ii here; if you plan to answer question ii on page 2 then answer question i here.

(i) Consider the trapezoid formula

$$T(\alpha,\beta,f)=\frac{f(\alpha)+f(\beta)}{2}(\beta-\alpha).$$

and the resulting quadrature method given by

$$\int_a^b f(x)dx \approx \sum_{j=0}^{N-1} T(x_j,x_{j+1},f)$$

where $N\in\mathbf{N}$ and $x_j=a+hj$ with $h=(b-a)/N$. Write a program to approximate the integral

$$\int_0^7 \frac{1}{2+\cos x}dx$$

for $N=16$ that prints the resulting approximation with 15 digits precision. Include the program and the output of the program in your submission.

(ii) Consider the differential equation given by

$$y'=\frac{1}{t^2}-\frac{y}{t}-y^2,\qquad y(1)=-1.$$

Approximate the solution to this differential equation on the interval $[1,2]$ using the RK4 method with $h=1/20$. Print the approximation of $y(2)$ and the error $\big|y(2)+1/2\big|$ to 15 digits precision. Include the program and the output of the program in your submission.

Please follow these instructions to submit your answer online. Create a subdirectory called `quiz2` in your home directory and place all programs and output in that directory. A `Makefile` is also recommended, but not necessary. Suppose `page1.c` contains the code you wrote to answer question 1 part ii on page 1 and you have compiled this program to create an executable called `page1`. To save the output of that program to a file, you may use a command such as

```
$ ./page1 >page1.out
```

Check that the output is what you expected, by printing the output in the terminal window using the command

```
$ cat page1.out
```

After you have worked the question from this page and the corresponding question from page 2, you should have two programs and two output files in the subdirectory as well as some optional stuff like a `Makefile`. At this point please submit your answers electronically before leaving with the commands

```
$ cd ..
$ submit quiz2
```

If prompted for a password, enter you NetID. If something goes wrong with the submission please let me know, and I'll find an alternative way for you to submit your files.

2. Answer one of the following two questions according to the following rule: If you answered question i on page 1 then answer question ii here; if you answered question ii on page 1 then answer question i here.

(i) Consider the 3-point Gaussian formula given by

$$G_3(\alpha,\beta,f) = \frac{\beta-\alpha}{2}\sum_{k=0}^{2} w_k f\Big(\alpha + \frac{\beta-\alpha}{2}(x_k+1)\Big)$$

where

$$\begin{array}{ccc} x_0 = -\sqrt{3/5}, & x_1 = 0, & x_2 = \sqrt{3/5}, \\ w_0 = 5/9, & w_1 = 8/9, & w_2 = 5/9, \end{array}$$

and the resulting quadrature method given by

$$\int_a^b f(x)dx \approx \sum_{j=0}^{N-1} G_3(x_j, x_{j+1}, f)$$

where $N \in \mathbf{N}$ and $x_j = a + hj$ with $h = (b-a)/N$. Write a program to approximate the integral

$$\int_0^4 \sin(e^x)\,dx$$

for $N = 32$ that prints the resulting approximation with 15 digits precision. Include the program and the output of the program in your submission.

(ii) Consider the differential equation given by

$$y' = \frac{y}{t} - \Big(\frac{y}{t}\Big)^2, \qquad y(1) = 1$$

Approximate the solution to this differential equation on the interval $[1,2]$ using Taylor's 3rd order method with $h = 1/24$. Print the approximation of $y(2)$ and the error $\big|y(2) - 2/(1+\log 2)\big|$ to 15 digits precision. Include the program and the output of the program in your submission.

3. [Extra Credit] Consider the heat equation with $C = 0.02$ given by

$$u_t = C(u_{xx} + u_{yy})$$

on the domain $[0,4]\times[0,4]$ with boundary conditions

$$u(t,0,y) = u(t,x,4) = 1 \qquad \text{and} \qquad u(t,4,y) = u(t,x,0) = 0$$

and initial condition

$$u(0,x,y) = \begin{cases} 1 & \text{if } \max\big\{|x-2|, |y-2|\big\} \le 1 \\ 0 & \text{otherwise} \end{cases}$$

for $(x,y) \in (0,4)\times(0,4)$. Find the value of u at time $t = 2$ given by $u(2,x,y)$ and plot the resulting heat distribution. Print your approximations of $u(2,1,3)$ and $u(2,1,1,)$ to 4 digits precision.

After finishing the problems on this page, please follow the instructions at the bottom of the first page to submit your answers online.