

```
> restart;
> ynp1:=a0*y(tn)+a1*y(tn-h)+a2*y(tn-2*h)+
      h*(bm1*D(y)(tn+h)+b0*D(y)(tn)+b1*D(y)(tn-h)+
      b2*D(y)(tn-2*h))+E5*h^5*(D@@5)(y)(theta)/5!;
```

$$y_{np1} := a_0 y(tn) + a_1 y(tn-h) + a_2 y(tn-2h) + h(bm_1 D(y)(tn+h) + b_0 D(y)(tn) + b_1 D(y)(tn-h) + b_2 D(y)(tn-2h)) + \frac{E_5 h^5 D^{(5)}(y)(\theta)}{120} \quad (1)$$

```
> r:=y(tn+h)-ynp1;
r := y(tn+h) - a0*y(tn) - a1*y(tn-h) - a2*y(tn-2h) - h*(bm1*D(y)(tn+h)
+ b0*D(y)(tn) + b1*D(y)(tn-h) + b2*D(y)(tn-2h)) - \frac{E5 h^5 D^{(5)}(y)(\theta)}{120}
```

```
> eq[0]:=eval(subs(y=(x->1),r));
eq0 := 1 - a0 - a1 - a2 \quad (3)
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```
> for j from 1 to 5 do
  tmp[j]:=eval(subs(y=(x->x^j),r));
  eq[j]:=coeff(tmp[j],h^j);
  print(eq[j]);
od:
      1 + a1 + 2 a2 - bm1 - b0 - b1 - b2
      1 - a1 - 4 a2 - 2 bm1 + 2 b1 + 4 b2
      1 + a1 + 8 a2 - 3 bm1 - 3 b1 - 12 b2
      1 - a1 - 16 a2 - 4 bm1 + 4 b1 + 32 b2
      1 + a1 + 32 a2 - 5 bm1 - 5 b1 - 80 b2 - E5 \quad (4)
```

```
> S1:=solve({seq(eq[k],k=0..5)},{a0,bm1,b0,b1,b2,E5});
S1 := { E5 = -\frac{19}{6} + \frac{11 a1}{6} - \frac{4 a2}{3}, a0 = 1 - a1 - a2, b0 = \frac{19}{24} + \frac{13 a1}{24} + \frac{a2}{3}, b1
= -\frac{5}{24} + \frac{13 a1}{24} + \frac{4 a2}{3}, b2 = \frac{1}{24} - \frac{a1}{24} + \frac{a2}{3}, bm1 = \frac{3}{8} - \frac{a1}{24} } \quad (5)
```

```
> method:=subs(E5=0,ynp1);
method := a0*y(tn) + a1*y(tn-h) + a2*y(tn-2h) + h*(bm1*D(y)(tn+h)
+ b0*D(y)(tn) + b1*D(y)(tn-h) + b2*D(y)(tn-2h)) \quad (6)
```

```
> m2:=subs(S1,method);
m2 := (1 - a1 - a2) y(tn) + a1 y(tn-h) + a2 y(tn-2h) + h \left( \left( \frac{3}{8} - \frac{a1}{24} \right) D(y)(tn+h) + \left( \frac{19}{24} + \frac{13 a1}{24} + \frac{a2}{3} \right) D(y)(tn) + \left( -\frac{5}{24} + \frac{13 a1}{24} + \frac{4 a2}{3} \right) D(y)(tn-h) + \left( \frac{1}{24} - \frac{a1}{24} + \frac{a2}{3} \right) D(y)(tn-2h) \right) \quad (7)
```

```
> m3:=eval(subs(D(y)=(x->f(x,y(x))),m2)); \quad (8)
```

$$\begin{aligned}
m3 := & (1 - a1 - a2) y(tn) + a1 y(tn - h) + a2 y(tn - 2 h) + h \left( \left( \frac{3}{8} - \frac{a1}{24} \right) A y(tn \right. \\
& + h) + \left( \frac{19}{24} + \frac{13 a1}{24} + \frac{a2}{3} \right) A y(tn) + \left( -\frac{5}{24} + \frac{13 a1}{24} + \frac{4 a2}{3} \right) A y(tn - h) \\
& \left. + \left( \frac{1}{24} - \frac{a1}{24} + \frac{a2}{3} \right) A y(tn - 2 h) \right)
\end{aligned} \tag{8}$$

**> f := (xi, eta) -> A \* eta;**

$$f := (\xi, \eta) \mapsto A \cdot \eta \tag{9}$$

**> m4 := y(tn+h) = m3;**

$$\begin{aligned}
m4 := & y(tn + h) = (1 - a1 - a2) y(tn) + a1 y(tn - h) + a2 y(tn - 2 h) + h \left( \left( \frac{3}{8} \right. \right. \\
& \left. \left. - \frac{a1}{24} \right) A y(tn + h) + \left( \frac{19}{24} + \frac{13 a1}{24} + \frac{a2}{3} \right) A y(tn) + \left( -\frac{5}{24} + \frac{13 a1}{24} \right. \right. \\
& \left. \left. + \frac{4 a2}{3} \right) A y(tn - h) + \left( \frac{1}{24} - \frac{a1}{24} + \frac{a2}{3} \right) A y(tn - 2 h) \right)
\end{aligned} \tag{10}$$

**> ceq := eval(subs(y=(s->rho^s), m4));**

$$\begin{aligned}
ceq := & \rho^{tn+h} = (1 - a1 - a2) \rho^{tn} + a1 \rho^{tn-h} + a2 \rho^{tn-2 h} + h \left( \left( \frac{3}{8} \right. \right. \\
& \left. \left. - \frac{a1}{24} \right) A \rho^{tn+h} + \left( \frac{19}{24} + \frac{13 a1}{24} + \frac{a2}{3} \right) A \rho^{tn} + \left( -\frac{5}{24} + \frac{13 a1}{24} \right. \right. \\
& \left. \left. + \frac{4 a2}{3} \right) A \rho^{tn-h} + \left( \frac{1}{24} - \frac{a1}{24} + \frac{a2}{3} \right) A \rho^{tn-2 h} \right)
\end{aligned} \tag{11}$$

**> ceq2 := subs({a1=1, a2=1, tn=1, h=1}, ceq);**

$$ceq2 := \rho^2 = -\rho + 1 + \frac{1}{\rho} + \frac{A\rho^2}{3} + \frac{5A\rho}{3} + \frac{5A}{3} + \frac{A}{3\rho} \tag{12}$$

**> S2 := solve(ceq2, rho);**

$$S2 := -1, -\frac{2A - \sqrt{3A^2 + 9}}{A - 3}, -\frac{2A + \sqrt{3A^2 + 9}}{A - 3} \tag{13}$$