In[1]:=
```
(* Question 1 *)
dp = Function[{f, g}, Integrate[f * g * Exp[-x ^ 2], {x, -Infinity, Infinity}]]
nm = Function[f, Sqrt[dp[f, f]]]
```

Out[1]= Function$\left[\{f, g\}, \int_{-\infty}^{\infty} f\, g\, \mathrm{Exp}\left[-x^2\right] dx\right]$

Out[2]= Function$\left[f, \sqrt{\mathrm{dp}[f, f]}\right]$

In[3]:=
```
N0 = 5
For[k = 0, k <= N0, k ++,
    w[k] = x ^ k;
    For[j = 0, j < k, j ++,
        w[k] = w[k] - dp[H[j], x ^ k] * H[j]];
    H[k] = Simplify[w[k] / nm[w[k]]];
    Print["H[", k, "]=", H[k]]]
(* The desired orthogonal polynomials are below *)
```

Out[3]= 5

H[0]= $\frac{1}{\pi^{1/4}}$

H[1]= $\frac{\sqrt{2}\, x}{\pi^{1/4}}$

H[2]= $\frac{\sqrt{2}\left(-\frac{1}{2} + x^2\right)}{\pi^{1/4}}$

H[3]= $\frac{x\left(-3 + 2 x^2\right)}{\sqrt{3}\, \pi^{1/4}}$

H[4]= $\frac{3 - 12 x^2 + 4 x^4}{2 \sqrt{6}\, \pi^{1/4}}$

H[5]= $\frac{x\left(15 - 20 x^2 + 4 x^4\right)}{2 \sqrt{15}\, \pi^{1/4}}$