

In[1]:= (* Question 4 *)

ynp1 = y[tn] + h / 2 * (f[tn, y[tn]] + f[tn + h, y[tn + h]])

f = Function[{t, y}, A * y]

method = y[tn + h] == ynp1

Out[1]= $\frac{1}{2} h (f[tn, y[tn]] + f[h + tn, y[h + tn]]) + y[tn]$

Out[2]= Function[{t, y}, A y]

Out[3]= $y[h + tn] == y[tn] + \frac{1}{2} h (A y[tn] + A y[h + tn])$

In[4]:= ceq = method /. y -> Function[s, rho ^ s]

ceq2 = ceq /. {tn -> 0, h -> 1}

S = Solve[ceq2, rho]

Out[4]= $\rho^{h+tn} == \rho^{tn} + \frac{1}{2} h (A \rho^{tn} + A \rho^{h+tn})$

Out[5]= $\rho == 1 + \frac{1}{2} (A + A \rho)$

Out[6]= $\left\{ \left\{ \rho \rightarrow \frac{-2 - A}{-2 + A} \right\} \right\}$

In[7]:= (* the linear stability region is all values of A such that |rho| < 1 *)

Z1t = Abs[rho] /. S[[1]]

Z1 = Z1t /. {A -> a + I * b}

Out[7]= $\text{Abs} \left[\frac{-2 - A}{-2 + A} \right]$

Out[8]= $\text{Abs} \left[\frac{-2 - a - i b}{-2 + a + i b} \right]$

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In[9]:= (* the red area is linearly stable, the blue unstable *)  
ContourPlot[Z1, {a, -2, 2}, {b, -2, 2}, Contours -> {1},  
ContourShading -> {Red, Blue}, PlotRange -> All, Exclusions -> None]
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