

Math 701 Exam 1 Version A

1. Given the lemmata

**Lemma 1.** The matrix  $I - vv^*$  is unitary if and only if  $\|v\|_2^2 = 2$  or  $v = 0$ .

**Lemma 2.** Let  $x$  and  $y$  be two vectors such that  $\|x\|_2 = \|y\|_2$  and  $\langle x, y \rangle$  is real. Then there exists a unitary matrix  $U$  of the form  $I - vv^*$  such that  $Ux = y$

Prove every square matrix is unitarily similar to a triangular matrix.

Math 701 Exam 1 Version A

*Answer to question 1 continued (if necessary).*

Math 701 Exam 1 Version A

2. Describe each of the following algorithms and state what the algorithm is used for:

(i) Newton's method.

(ii) Horner's method.

(iii) The shifted  $QR$  algorithm of Francis.



6. Explain what is meant by a subordinate matrix norm.

7. Let  $A \in \mathbf{C}^{n \times n}$  where  $n \in \mathbf{N}$ . Give formulae for the following matrix norms:

(i)  $\|A\|_\infty$

(ii)  $\|A\|_1$

8. Let  $f: \mathbf{R} \rightarrow \mathbf{R}$  be differentiable. The condition number of  $f$  is

(A)  $\frac{xf'(x)}{f(x)}$

(B)  $\frac{xf(x)}{f'(x)}$

(C)  $\frac{f'(x)}{xf(x)}$

(D)  $\frac{f(x)}{xf'(x)}$

(E) none of these.