Math 701 Quiz 2 Version A

INSTRUCTIONS: Complete 2 questions out of the 2 questions below. Clearly indicate which problems you wish graded. Work each problem on a separate sheet of paper (or more if necessary).

1. Let f be a function in $C^{n+1}[a,b]$ and let p be the polynomial of degree at most n that interpolates the function f at $n+1$ distinct points $x_0, x_1, \ldots, x_n$ in the interval $[a,b]$. Prove that to each x in $[a,b]$ there corresponds a point ξ_x in (a,b) such that

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi_x) \prod_{i=0}^{n} (x - x_i).$$

2. Let f be a function in $C^{n+1}[a,b]$. Prove for any x and c in the closed interval $[a,b]$

$$f(x) = \sum_{k=0}^{n} \frac{1}{k!} f^{(k)}(c)(x-c)^k + R_n(x)$$

where

$$R_n(x) = \frac{1}{n!} \int_c^x f^{(n+1)}(t)(x-t)^n dt.$$