Math 701 Quiz 1 Version A

1. Find a suitable trigonometric identity so that $1 - \cos x$ can be accurately computed for small x with calls to the system functions for $\sin x$ or $\cos x$.

2. State Taylor's theorem including all hypothesis and the remainder term.

3. State Newton's method.

4. Prove only one of the following:

(i) Taylor's theorem.

(ii) Let f be a twice continuously differentiable function and p be a point such that $f(p) = 0$ and $f'(p) \neq 0$. Prove that Newton's method is quadratically convergent provided x_0 is close enough to p.

Math 701 Quiz 1 Version A

Proof of Taylor's theorem or the quadratic convergence of Newton's method continues ...

5. Let $A \in \mathbf{R}^{d\times d}$ be a symmetric positive semidefinite matrix. Consider the

> **Power Method.** Choose $x_0 \in \mathbf{R}^d$ randomly. Then recursively compute $y_n = Ax_n$ and $x_{n+1} = y_n/\|y_n\|$ for $n \geq 0$.

Show for almost every choice of x_0 that the limits

$$\lambda = \lim_{n\to\infty} \|y_n\| \qquad \text{and} \qquad \xi = \lim_{n\to\infty} x_n$$

exist and that λ and ξ form an eigenvalue-eigenvector pair for A such that $A\xi = \lambda\xi$.

Proof of the convergence of the power method continues . . .

Math 701 Quiz 1 Version A

6. For $x \in \mathbf{R}^d$ define $\|x\|_p = \left(\sum_{k=1}^{d} |x_k|^p\right)^{1/p}$.

(i) Prove that $\|x\|_2 \leq \|x\|_1$.

(ii) [Extra credit] Prove or disprove that $\|x\|_p \leq \|x\|_1$ for every $p \geq 1$.