Math 713 Practice Quiz 1 Version A

1. Let $I_n = [-n, 1/n]$ for $n \in \mathbf{N}$ and $U = \bigcup_{n=1}^{\infty} I_n$. Then

(A) $U = (-\infty, 0)$

(B) $U = (-\infty, 0]$

(C) $U = (-\infty, 1)$

(D) $U = (-\infty, 1]$

(E) none of these

2. Let $J_n = [0, 1/n]$ for $n \in \mathbf{N}$ and $V = \bigcap_{n=1}^{\infty} J_n$. Then

(A) $V = \emptyset$

(B) $V = \{ 0 \}$

(C) $V = [0, 1)$

(D) $V = [0, 1]$

(E) none of these

3. Let $f \colon X \to Y$. If $A \subseteq X$ then

(A) $f(A) = \{ f(x) : x \in X \}$

(B) $f(A) = \{ x \in X : f(x) \in Y \}$

(C) $f(A) = \{ x \in X : f(x) \in A \}$

(D) $f(A) = \{ x \in X : f(x) \in Y \setminus A \}$

(E) none of these

4. Let Ω be a set and $\mathcal{A}$ be a collection of subsets of Ω such that $A \in \mathcal{A}$ implies $A^c \in \mathcal{A}$ and $A, B \in \mathcal{A}$ implies $A \cup B \in \mathcal{A}$. Then $\mathcal{A}$ must be

(A) an algebra

(B) a σ-algebra

(C) a monotone class

(D) both (A) and (B)

(E) both (A), (B) and (C)

5. [Extra Credit] State the first and last names of three world famous mathematicians either dead or alive who do not work at UNR. Correct spelling is essential.

6. A set $U \in \mathbf{R}$ is open if for every $x \in U$ there exists $r > 0$ such that $(x-r, x+r) \subseteq U$. This is equivalent to saying U is open if

(A) for every sequence $x_n \in U$ and $x \in \mathbf{R}$ then $x_n \to x$ implies $x \in U$

(B) for every sequence $x_n \in U^c$ and $x \in \mathbf{R}$ then $x_n \to x$ implies $x \in U^c$

(C) for every $x \in U^c$ there exists $r > 0$ such that $U^c \subseteq [x-r, x+r]$

(D) every sequence $x_n \in U$ has a convergent subsequence

(E) none of these

7. Let $D \subseteq \mathbf{R}$ and $f\colon D \to \mathbf{R}$. Suppose for every $\epsilon > 0$ there is $\delta > 0$ such that $a, b \in D$ and $|a-b| < \delta$ implies $|f(a) - f(b)| < \epsilon$. Then f must be

(A) continuous

(B) uniformly continuous

(C) differentiable

(D) both (A) and (B)

(E) both (A), (B) and (C)

8. Let x_n be a sequence of real numbers. A real number x is said to be a cluster point of x_n if for each $\epsilon > 0$ and $N \in \mathbf{N}$ there is an $n \geq N$ such that $|x - x_n| < \epsilon$. This is equivalent to saying $x \in \mathbf{R}$ is a cluster point of x_n if

(A) $x \in \overline{E}$ where $E = \{\, x_n : n \in \mathbf{N} \,\}$

(B) there exists a subsequence x_{n_k} such that $x_{n_k} \to x$

(C) there exists a subsequence x_{n_k} of distinct elements such that $x_{n_k} \to x$

(D) $x \in [\alpha, \beta]$ where $\alpha = \liminf x_n$ and $\beta = \limsup x_n$

(E) none of these

9. Let $D \subseteq \mathbf{R}$ and $f_n\colon D \to \mathbf{R}$ for $n \in \mathbf{N}$. Suppose for each $x \in D$ and $\epsilon > 0$ there is $N \in \mathbf{N}$ such that $n, m \geq N$ implies $|f_n(x) - f_m(x)| < \epsilon$. Then the sequence f_n of real valued functions must be

(A) pointwise convergent

(B) uniformly convergent

(C) differentiable

(D) both (A) and (B)

(E) both (A), (B) and (C)

10. Fill in the missing blanks in the statement of the following axiom.

Completeness Axiom. A $\boxed{}$ subset of real numbers that is bounded above has a $\boxed{}$.

11. Fill in the missing blanks in the statement of the following theorem.

Theorem 2.7. A $\boxed{}$ function on $[a, b]$ is Riemann integrable if and only if the set of points of discontinuity of the function has measure $\boxed{}$.

12. Show there is an irrational number between any two rational numbers.

13. Let $U \subseteq \mathbf{R}$ be a non-empty bounded open set and $x \in U$. Define

$$a = \inf\{\, y : y < x \text{ and } (y, x) \subseteq U \,\}.$$

Show that $a \notin U$.

14. Prove one of the following theorems.

(i) Let $D \subseteq \mathbf{R}$ and $f_n\colon D \to \mathbf{R}$ be a sequence of continuous functions. Suppose $f_n \to f$ uniformly. Then f is continuous.

(ii) Suppose $f\colon [0,1] \to \mathbf{R}$ is continuous and $f(c) > 0$ for some $c \in (0,1)$. Show there is $h > 0$ such that $|x - c| < h$ implies $f(x) > 0$.

15. Prove or find a counter example to one of the following claims.

(i) Let $x_n \in \mathbf{R}$ for $n \in \mathbf{N}$ and $h\colon \mathbf{N} \to \mathbf{N}$ be a bijection. Define $y_n = x_{h(n)}$. Let $E = \{\, x \in \mathbf{R} : x \text{ is a cluster point of } x_n \,\}$ and $F = \{\, y \in \mathbf{R} : y \text{ is a cluster point of } y_n \,\}$. Prove or find a counter example to the claim that $E = F$.

(ii) For $A, B \subseteq \mathbf{R}$ define $A \cdot B = \{\, ab : a \in A \text{ and } b \in B \,\}$. Prove or find a counter example to the claim that $\overline{A} \cdot \overline{B} = \overline{A \cdot B}$.