

M761 Partial Differential Equations I Review

Chapter 1

- *1. Existence, uniqueness and continuous dependence on the initial data for the ordinary differential equation

$$\begin{cases} y' = f(t, y) \\ y(0) = y_0 \end{cases}$$

where $y \in \mathbf{R}^n$ and $f : D \rightarrow \mathbf{R}^n$ where $D \subseteq \mathbf{R} \times \mathbf{R}^n$ is open and contains $(0, y_0)$ and is continuous in the first variable and uniformly Lipschitz in the second.

- 2. Definition of what it means for a partial differential equation to be well posed.
- 3. Proof of Gronwall's inequality.
- *4. Poincaré inequality and the energy inequality for the heat equation.
- 5. Separation of variables and Fourier series solution for the heat and wave equations.
- *6. Variational formulation of Laplace equation.
- 7. D'Alembert's solution of the wave equation.

Chapter 2

- *8. Multiindex notation.
- 9. Classification of elliptic, parabolic and hyperbolic equations.
- 10. Definition of the symbol of the principle part $L^p(x, i\xi)$.
- 11. Characteristics.
- 12. Statement of the Cauchy-Kovaleski theorem.
- *13. Definition of a real analytic function.
- *14. Characterization of real analytic functions in terms of C_r^M . Know the proofs for functions of one variable.

Chapter 3

- 15. What is a hyperbolic conservation law?
- *16. Solution of a first order single hyperbolic partial differential equation by the method of characteristics.
- 17. Definition of a k -Riemann invariant.
- 18. Definition of a weak solution to a hyperbolic conservation law.
- *19. Statement and derivation of the Rankin-Hugoniot condition.