Math 761 Additional Problems for Homework 2

1. Explicitly compute the 36 entries of the matrix $\mathbf{W}_6$ corresponding to the discrete Fourier transform $\hat{x} = \mathbf{W}_6\, x$ where $\hat{x}$ and x are vectors of length 6 given by

$$\hat{x} = \begin{bmatrix} \hat{x}_0 \\ \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_5 \end{bmatrix}, \qquad x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_5 \end{bmatrix} \qquad \text{and} \qquad \hat{x}_n = \sum_{j=0}^{5} x_j e^{-i2\pi jn/6}.$$

2. Show that $\mathbf{W}_6$ can be factored as

$$\mathbf{W}_6 = \begin{bmatrix} I_3 & \Omega_3 \\ I_3 & -\Omega_3 \end{bmatrix} \begin{bmatrix} \mathbf{W}_3 & 0 \\ 0 & \mathbf{W}_3 \end{bmatrix} P_6$$

   where I_3 is the identity matrix, Ω_3 is diagonal, $\mathbf{W}_3$ is the matrix corresponding to the discrete Fourier transform of length 3 and P_6 is a permutation matrix. Explicitly write out $\mathbf{W}_3$, Ω_3 and P_6.

3. Show that $\mathbf{W}_6$ can be factored as

$$\mathbf{W}_6 = \begin{bmatrix} I_2 & \mathrm{X}_2 & \Psi_2 \\ I_2 & c\mathrm{X}_2 & c^2\Psi_2 \\ I_2 & c^2\mathrm{X}_2 & c\Psi_2 \end{bmatrix} \begin{bmatrix} \mathbf{W}_2 & 0 & 0 \\ 0 & \mathbf{W}_2 & 0 \\ 0 & 0 & \mathbf{W}_2 \end{bmatrix} Q_6$$

   where I_2 is the identity matrix, X_2 and Ψ_2 are diagonal, c is a complex constant, $\mathbf{W}_2$ is the matrix corresponding to the discrete Fourier transform of length 2 and Q_6 is a permutation matrix. Explicitly write out $\mathbf{W}_2$, X_2, Ψ_2, c and Q_6.