Math 761 Quiz 2 Version A

1. Let $\gamma(t) = e^{2\pi i t}$ where $t \in [0,1]$ Use Cauchy's integral formula to evaluate the following integrals:

(i) $\displaystyle\int_{[\gamma]} \frac{\sin\zeta}{2\zeta+1}\, d\zeta$

(ii) $\displaystyle\int_{[\gamma]} \frac{\sin\zeta}{\zeta+2}\, d\zeta$

(iii) $\displaystyle\int_{[\gamma]} \frac{\sin\zeta}{2\zeta^2+1}\, d\zeta$

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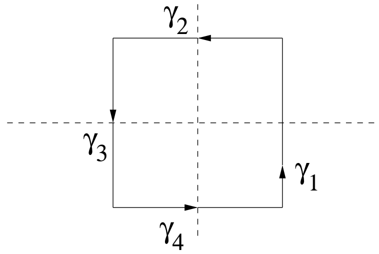
2. Consider the curve given by $C = [\gamma_1] + [\gamma_2] + [\gamma_3] + [\gamma_4]$ where $\gamma_1(t) = 1 + i(2t - 1)$, $\gamma_2(t) = 1 - 2t + i$, $\gamma_3(t) = -1 + i(1 - 2t)$ and $\gamma_4(t) = 2t - 1 - i$.



Find $\displaystyle\int_C \frac{i\cos(\zeta)}{\zeta^3 - 2\zeta^2}\, d\zeta.$

3. Prove if f is bounded on $[0,1]$ that $\langle f, h_{j,k}\rangle = \mathcal{O}(2^{-j/2})$ as $j \to \infty$.

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4. Prove one of the following:

(i) If $x_0 \in [0,1]$ is a dyadic irrational then there is an increasing sequence j_n of natural numbers such that

$$\frac{1}{4} \le 2^{j_n} x_0 \bmod 1 \le \frac{1}{2}.$$

(ii) Let $h_{j,k}$ be the functions which make up the Haar system. Define

$$a = \frac{k}{2^j}, \qquad b = \frac{k+1}{2^j} \qquad \text{and} \qquad c = \frac{a+b}{2}.$$

If $x_0 \in [a,c)$ then

$$\int_a^{x_0} 5h_{j,k}(t)\,dt + \int_{x_0}^b h_{j,k}(t)\,dt = 2^{2+j/2}(x_0 - a).$$