

Math 762 Quiz 1 Version A

1. Suppose $f \in C^0 \cap L^1$ and $\hat{f} \in L^1$. Prove any two of the following three statements:

(i)
$$\lim_{\tau \rightarrow 0^+} \frac{1}{\tau} \int_{\mathbf{R}} f(t) e^{-\pi(x-t)^2/\tau^2} dt = f(x).$$

(ii)
$$\frac{1}{\tau} \int_{\mathbf{R}} f(t) e^{-\pi(x-t)^2/\tau^2} dt = \int_{\mathbf{R}} \hat{f}(\gamma) e^{-\pi\tau^2\gamma^2} e^{2\pi i\gamma x} d\gamma.$$

(iii)
$$\lim_{\tau \rightarrow 0^+} \int_{\mathbf{R}} \hat{f}(\gamma) e^{-\pi\tau^2\gamma^2} e^{2\pi i\gamma x} d\gamma = \int_{\mathbf{R}} \hat{f}(\gamma) e^{2\pi i\gamma x} d\gamma.$$