

M119 Sample Final Exam, Fall 2002

Last name:

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First name:

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Student ID:

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Instructor:

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Directions:

- This exam consists of 18 multiple choice questions and 2 partial credit problems.
- For each multiple choice question, you must circle the letter on this page that corresponds to the **best** choice. Each multiple choice question is worth 4.5 points.
- For each partial credit problem you must show all of your work in the space provided and circle your answers.

1) A B C D E

10) A B C D E

2) A B C D E

11) A B C D E

3) A B C D E

12) A B C D E

4) A B C D E

13) A B C D E

5) A B C D E

14) A B C D E

6) A B C D E

15) A B C D E

7) A B C D E

16) A B C D E

8) A B C D E

17) A B C D E

9) A B C D E

18) A B C D E

Do not write below this line

multi	_____	points
	×4.5 =	
=====	partial	
=====	total	

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
4.5	9	13.5	18	22.5	27	31.5	36	40.5	45	49.5	54	58.5	63	67.5	72	76.5	81

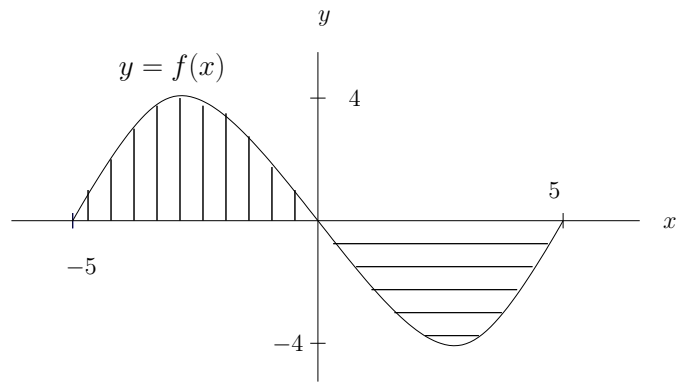
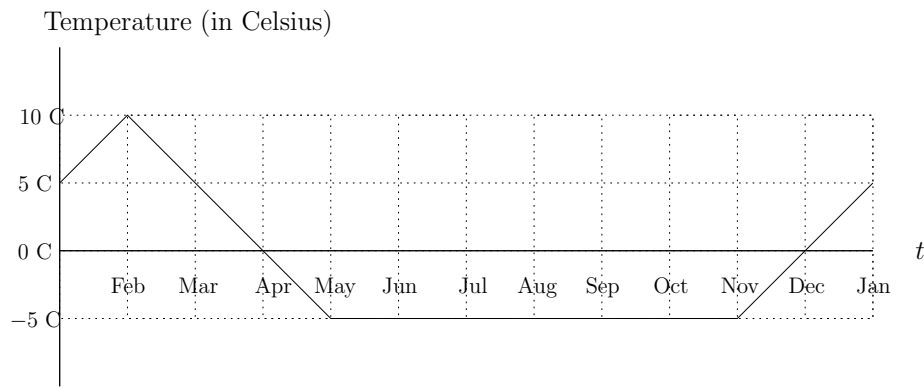


FIGURE 1. Two regions between curve $y = f(x)$ and x -axis.

- (1) Bounded between a curve $y = f(x)$ and the x -axis are the two regions pictured above. If the area of each region is 15, then $\int_{-5}^5 f(x) dx$ equals
- 30
 - 0
 - 20
 - 15
- (2) An antiderivative for $f(x) = e^x$ is
- e^x
 - $x \cdot e^x$
 - e^{-x}
 - does not exist
 - None of the above is correct.
- (3) The indefinite integral $\int x^3 dx$ equals
- $\frac{1}{4}x^4$
 - x^4
 - $\frac{1}{4}x^4 + C$
 - $4x^4 + C$
- (4) The value of the definite integral $\int_1^2 \frac{1}{x^4} dx$ is
- $\frac{7}{8}$
 - $\frac{7}{24}$
 - $-\frac{7}{24}$
 - $-\frac{7}{8}$
 - none of the above.



- (5) Scientists have discovered an organism living in Antarctica whose cells freeze but survive upon thawing. The year round body temperature of one of these organisms is given in Figure 2. The average value of the body temperature over the entire year
- equals 0 Celsius
 - equals 2.5 Celsius
 - equals 5 Celsius
 - is negative
- (6) A helicopter is rising straight up. Its vertical velocity after t seconds is $4t + 1$ feet per second. What is the total change in the helicopter's height during the first 5 seconds?
- 5 feet
 - 25 feet
 - 55 feet
 - 75 feet

- (7) The population, in millions, of a state, t years since 1950 is given by

$$P(t) = \frac{50}{1 + 1.82e^{-0.03t}}.$$

The population is growing the fastest in

- 1970
 - 1960
 - 1980
 - 1965
 - 1985
- (8) Let $h(x) = f(g(x))$ where f and g are two differentiable functions satisfying $g(2) = 1$, $g'(2) = 6$, $f(1) = 2$, and $f'(1) = 3$. Then which of the following *must* be true?
- $h'(1) = 18$
 - $h(1) = 1$
 - $h'(2) = 18$
 - $h(2) = 1$
 - None of the above

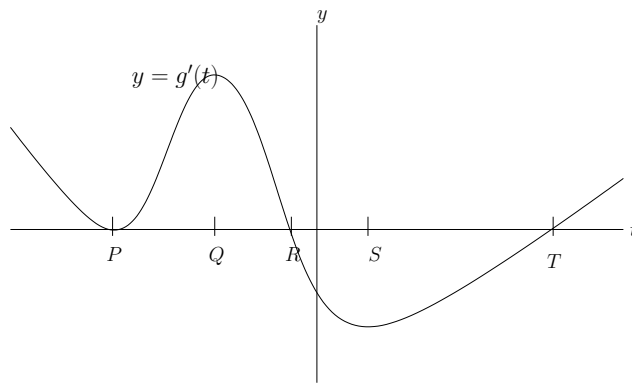


FIGURE 2. The graph of $g'(t)$.

- (9) The graph of the **derivative** of the function g appears above. Choose the **one best** answer.

- A. The function g has a local maximum at $t = T$.
- B. P corresponds to both a critical point and an inflection point of the function g .
- C. The function g has a local minimum at $t = R$.
- D. S corresponds to both a critical point and an inflection point of the function g .
- E. None of the above is correct.

- (10) Let k be a constant. The derivative of $\ln(kx)$ equals

- A. $\frac{1}{x} + \frac{1}{k}$
- B. $\frac{1}{kx}$
- C. $\frac{1}{x}$
- D. $\frac{k}{x}$
- E. none of the above

- (11) The amount, $P(t)$, of a certain radioactive substance is known to decay exponentially with time t . Data collected on this decay is presented in the following table.

t (in years)	10	20	30	40
$P(t)$ (in grams)	6.10		3.00	2.10

The missing entry in the table is approximately

- A. 4.06 grams
- B. 4.34 grams
- C. 4.13 grams
- D. 4.27 grams
- E. 3.86 grams

- (12) If the graph of a function $f(x)$ is concave up, then the slope of the line that is tangent to the graph at $(x, f(x))$

A. is always positive
 B. is always negative
 C. increases as x increases
 D. decreases as x increases
 E. none of the above

- (13) A man drinks a cup of coffee containing about 100 mg of caffeine. Every hour, the amount of caffeine remaining in the man's body decreases by 17%. In 5 hours, the amount still present is

A. $100(.17)^5$
 B. $100e^{-.17 \cdot 5}$
 C. $100(1.17)^5$
 D. $100(.83)^5$
 E. None of the above.

- (14) Human error leads to the opening of a valve that releases radioactive gas into the air over a 3 second interval. The table below shows the rate at which the gas escaped through the valve t seconds after the valve was opened.

t (in seconds)	0	1	2	3
$S'(t)$ (in m^3/sec)	4.2	3.5	2.7	1.3

Assuming that the rate is decreasing, which of the following represents the best upper estimate on the cumulative amount of gas released in those 3 seconds?

A. 7.5 m^3
 B. 10.4 m^3
 C. 11.7 m^3
 D. 12.6 m^3
 E. None of the above.

- (15) Let g be the function given in the table below.

t	0.0	0.2	0.4	0.6
$g(t)$	6.2	3.1	2.2	1.1

By taking the average of two average rates of change, find the best estimate of $g'(0.4)$.

A. -2.0
 B. -4.0
 C. -5.0
 D. -10.0
 E. None of the above.

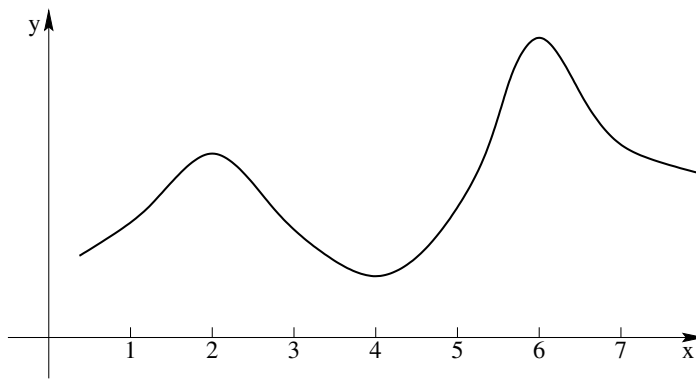


FIGURE 3. A curve that is the graph of a certain type of function.

(16) The curve pictured above could be the graph of

- A. an exponential function
- B. a polynomial
- C. the natural logarithm
- D. a power function
- E. a linear function

(17) The indefinite integral

$$\int \frac{x}{x^2 + 4} dx$$

equals

- A. $(x^2 + 4)^{-2} + C$
- B. $-(x^2 + 4)^{-1} + C$
- C. $\frac{1}{2} \ln(x^2 + 4) + C$
- D. $-\frac{1}{2}(x^2 + 4)^{-2} + C$
- E. None of the above.

(18) An investment account earns a constant annual interest rate of 7% and is compounded continuously. At approximately what constant rate should you deposit money in the account so that after 5 years the account holds \$50,000?

- A. \$7,288 per year
- B. \$8,351 per year
- C. \$10,000 per year
- D. \$35,234 per year
- E. None of the above.

Partial credit problems.
Show all work! Circle your answers!

(19) The owner of a 300 seat movie theatre would sell every seat if she were to charge \$4 per seat. For each \$.50 increase in the price of the seat she sells 10 fewer seats.
(Each part is worth 2 points)

A. Write the number of seats sold, q , as a function of the price p .

B. Write the movie theatre's revenue R as a function of the price p alone.

C. Find the derivative, $R'(p)$.

D. Find the critical points of $R(p)$.

E. What is the price p that maximizes the movie theatre's revenue R .

(20) The rate of change in the population of a bat colony is given by $r(t) = 8 \cdot \sqrt{t} + 6$ where t is the number of years since the colony was discovered. (*Each part is worth 2 points*)

A. Use a left hand sum with 3 equal subintervals to estimate the total change in the colony's population during the 6 years that followed its discovery.

B. Find an antiderivative of the function $r(t)$.

C. Find the total change in the population of the colony during the 6 years that followed its discovery.